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Choice of item pricing feedback schemes for multiple unit reverse combinatorial auctions

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Recently, interest in combinatorial auctions has extended to include trade in multiple units of heterogeneous items. Combinatorial bidding is complex and iterative auctions are used to allow bidders to sequentially express their preferences with the aid of auction market information provided in the form of price feedbacks. There are different competing designs for the provision of item price feedbacks; however, most of these have not been thoroughly studied for multiple unit combinatorial auctions. This paper focuses on addressing this gap by evaluating several feedback schemes or algorithms in the context of multiple unit auctions. We numerically evaluate these algorithms under different scenarios that vary in bidder package selection strategies and in the degree of competition. We observe that auction outcomes are best when bidders use a naïve bidding strategy and competition is strong. Performance deteriorates significantly when bidders strategically select packages to maximize their profit. Finally, the performances of some algorithms are more sensitive to strategic bidding than others.

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1. Introduction

Combinatorial auction designs, which allow bidding on single as well as packages of items, have been used to trade different types of goods and services. These auctions increase the flexibility with which bidders express their choices. Bidders benefit from the opportunity to exploit synergies between package items while the auctioneer benefits from increased competition. Recently, these auctions have been tested for multiple unit cases, where bidders are able to combine several items or services each of which can be offered at different levels. Potential application areas for the auctions are many and include agriculture, conservation and development planning.

However, the wide strategy space in a combinatorial auction can make bidding complex. Multiple unit combinatorial auctions are best conducted iteratively as a series of rounds where bidders can progressively reveal their preferences through a sequence of revised bids. The bidding process can then be facilitated through the provision of pricing information to bidders based on intermediate round bidding results. Often this pricing feedback is provided in the form of anonymous linear or item prices (hereafter termed item prices), where bundle prices are equal to the value of the package calculated at the item prices (Parkes, 2006). The item prices data are *anonymous* because they are the same for all bidders. Different algorithms for computing item price feedbacks have been proposed (Pikovsky, 2008). Some of the notable algorithms are the Resource Allocation Design or RAD (Kwasnica *et al*, 2005), the smoothed anchoring (SmAnch) (Hoffman, 2006), the Data Envelopment Analysis (DEA) based pricing algorithm (Aparicio *et al*, 2008) and nucleolus algorithms (Dunford *et al*, 2007).

All of these algorithms are interesting, each with their own particular advantages. However, these schemes (excepting the DEA-based algorithm) have not yet been tested for cases where the items or services auctioned can be offered at different levels. It should be noted here that there are alternative item price feedback-based mechanisms such as Clock Combinatorial auctions and ALPSm, which have been tested in a wide range of scenarios (Pikovsky, 2008). However, to manage the scope of our work we have concentrated only on selected algorithms that have not been tested for multiple unit reverse combinatorial auctions. Such multiple unit combinatorial auctions could significantly increase the complexity of the package selection problem for bidders, since bidders have to take into account not only the economies of scope (cost synergies) among different services but also the economies of scale arising from providing items/services at different possible levels. For example, if a bidder is capable of providing up to three services/items and each of those services could be

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provided at three levels (including an option of not offering the item), this bidder has 26 ($3^3-1=26$) potential bundles or packages that he/she should offer. That is, the selection of suitable bids could be a complex and expensive task for the bidder. Therefore, the success of a price feedback algorithm depends on how effectively it provides market information to guide the bidding towards the final allocation.

We evaluate price feedback algorithms in auction market scenarios that vary in two important aspects, namely, bidder package selection rules and the degree of competition. The package selection strategy determines the capacity of the bidder to process market signals and determines the type and number of bids submitted by a bidder. On the other hand, it is expected that auction efficiency would increase with competition. This study aims to provide information on price feedback designs that perform consistently against different package selection strategies and competition scenarios. This knowledge would help auctioneers to choose designs that improve auction outcomes.

In the next section, we describe iterative combinatorial auctions, including the winner determination problems and the provision of price feedbacks. We then present the details of our numerical experiments, including the structure of the packages offered, the package selection strategies used by bidders, the competition scenarios explored and the performance measures employed. Section 4 presents and discusses the results. The paper is summarized and conclusions drawn in Section 5.

2. Iterative combinatorial auctions

In natural resource management and in many other procurement problems, multiple unit combinatorial auctions are more relevant than auctions for unique items. For example, a landholder is capable of undertaking conservation activities to conserve different population sizes of different species. That is, different bids from a landholder could offer different levels of conservation services for a given species, possibly in combination with services that benefit another species. The objective of the auctioneer is to select the bids that would meet the conservation target while minimizing procurement costs. This selection problem is known as the winner determination problem (WDP) and is formally described below.

Let us assume that the auctioneer is aiming to purchase multiple units of several items/services, say $U=\{u_1, u_2, \ldots, u_g\}$, $u_k \in \Re^+$. Assume that there are N bidders, $\{1, 2, \ldots, N\}$, participating in the auction. Each bidder submits a set of bids, $A_i=\{A_{i1}, A_{i2}, \ldots, A_{im}\}$, where jth bid from bidder i, A_{ij}, conveys the information on the bid price p_{ij} and the set of individual items $(\lambda_{ij}^1, \lambda_{ij}^2, \ldots, \lambda_{ij}^g)$ offered, with $\lambda_{ij}^k \geqslant 0$ denoting the number of units of item k included in the bid. The auctioneer's WDP is the following constrained cost minimization problem:

$$\begin{aligned} Z = \min \sum_{i=1}^{N} \sum_{j=1}^{m} p_{ij} x_{ij} & \\ \text{s.t.} \quad \sum_{i,j} \lambda_{ij}^k x_{ij} \geqslant u_k & \\ \sum_{i} x_{ij} \leqslant 1 & \\ x_{ij} \in \{0, 1\} & \qquad \text{(WDP 01)} \end{aligned}$$

where Z is the minimized procurement cost of meeting the purchase target U, and x_{ij} is a binary variable indicating whether bid j from bidder i has been selected ($x_{ij}=1$) or not ($x_{ij}=0$). The first constraint states that the selected set of bids collectively satisfy or meet the auctioneer's target demand. The second constraint, $\sum_i x_{ij} \leqslant 1$, ensures that at most one package is purchased from each bidder. An auction with this feature is said to be an XoR bidding (Xia *et al*, 2004). The third constraint ensures that there is no fractional selection, that is, a bid is successful in its entirety or not at all.

Bidders face the challenge of expressing their preferences for different combinations of items. Iterative formats lessen some of the preference elicitation problem by allowing bidders to incrementally reveal their choices during the course of the auction (Aparicio *et al*, 2008). In a round-based iterative auction bidding opportunities are provided in the form of distinct rounds. During a round, a set of bids are first submitted. The auctioneer then computes the WDP results. The auctioneer also runs a feedback price computation algorithm. These price feedbacks are then provided to the bidders as indications of item prices implied by the bids submitted in the previous round. A new round is then started and, subject to some activity rules, bidders revise their bids and submit offers again. The auction ends when a termination rule is satisfied and a final allocation is made with winners in this last round obtaining contracts. The termination rule could be based on a maximum number of rounds.

In this paper, we are interested in price feedbacks given as item prices, rather than as a bundle or as package prices, since item prices can be easily adapted to multiple unit auctions and are also easier to understand for bidders. A compatible set of item prices should explain or rationalize results from the WDP allocation. That is, ideally, the item prices provided as feedback would be such that the computed values of winning (losing) packages are not smaller (greater) than their respective bid prices (Iftekhar *et al*, 2011). However, it has been shown that a set of prices could be compatible with the solution from the WDP if and only if integer requirements are redundant, that is, if the LP relaxation of the problem generates an integer solution (Bikhchandani and Mamer, 1997). Since combinatorial auctions mostly deal with complementary items, it can be difficult to find a set of compatible prices that rationalize the WDP solution as described above. Therefore, different post-processing heuristics or item price calculation algorithms have been designed with the goal of approximating the solution from the WDP as closely as possible.

We focus on a closely related set (six) of anonymous linear price feedback algorithms that approximate market clearing prices. These include the linear and nonlinear variants of RAD, the smoothed anchoring algorithm, two nucleolus-based algorithms, and a DEA-based algorithm. The mathematical details of these algorithms are provided in Appendix A. Price information produced by these algorithms reflects the bidding strategies adopted by bidders in the previous round. As a consequence, bidders have the potential to influence both the market allocation and the information feedbacks by bidding strategically (Kagel *et al*, 2010). We test the performances of the price feedback algorithms for different bidding strategies. Where only a single activity rule is implemented, that is, during the intermediate rounds, the provisional winning bids are carried forward to the next round. Results from this study would be useful in understanding bidders' strategic behaviour when more rigid market structure (such as clock combinatorial auction) are tested in future studies on combinatorial conservation auctions.

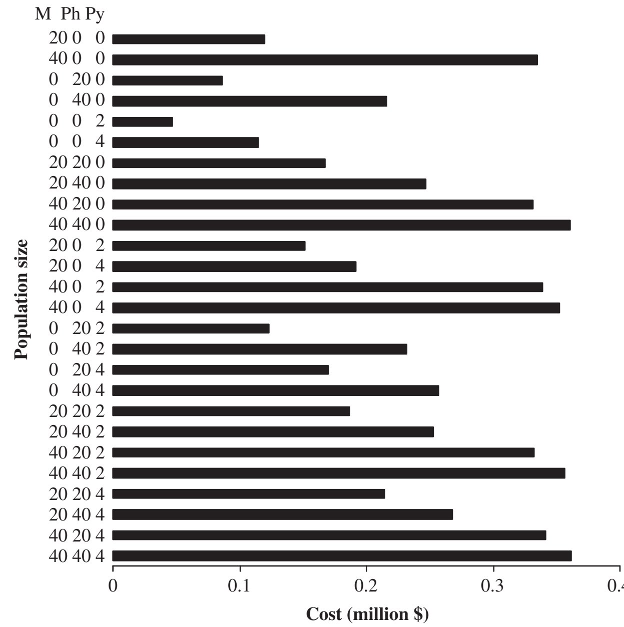

Figure 1 Conservation costs for 26 different combinations of malleefowl (M), phascogale (Ph) and python (Py) population outcomes. Conservation costs (x-axis) and population outcomes (y-axis) are defined over a 10-year planning horizon.

3. Computational experiments

The structure of our experiments is motivated primarily by conservation auctions, where landholders submit bids to win contracts to undertake projects that deliver conservation services. The key features of our numerical experiments are described below.

3.1. Package structure

There are a range of intervention activities that a bidder can undertake to obtain a certain level of conservation outcome. The activities can be combined in different ways and the cost of achieving an outcome depends on how optimally these interventions are chosen and combined. We use a bioeconomic model developed by Iftekhar *et al* (2009) to determine optimal cost functions for bidders. The model estimates optimal costs for conserving different population sizes of three endangered native species (red-tailed phascogale (Ph), carpet python (Py) and malleefowl (M)) that are found in the wheatbelt region of Western Australia. For each species we have considered three possible levels of population size outcomes. A landholder could propose packages on any combination of these levels. Therefore, a landholder has 26 possible packages that they could offer (ie, 3^3-1). The population outcomes levels as well as the optimal bidder costs are shown in Figure 1.

3.2. Bidder package selection strategies

How should a bidder determine the packages he/she offers for the next round? Different kinds of package selection strategies have been explored in the literature. For example, in the context of combinatorial auctions for lane procurement by truckload transportation service providers, Song and Regan (2003) developed a model that uses optimization-based methods to select lanes on which to bid. Later, Lee *et al* (2007) proposed another carrier optimization model and their results indicate that carriers can benefit by using optimization-based lane selection methods. An *et al* (2005) observed bidding behaviour in real-world single round transportation auctions, where carrier companies bid for contracts on lanes in a transportation network. They observed that bidders employed some alternative package selection strategies, including: (1) select only individual lanes, (2) combine only high value lanes, (3) select lanes competitively, (4) mix up high and low value lanes and (5) bundle only lanes with geographic proximity. Motivated by such observations, in their simulations they used three bundle selection strategies that correspond to strategies (1), (2) and (3) above, respectively, and which they refer to as Naive Strategy, Internal-Based Strategy (INT), and Competition-Based Strategy (COMP). In an experimental study, Park and Rothkopf (2001) observed that many bidders try to manipulate competition by withholding bids that are complementary to the existing bids of their competitors. Bidders with high synergy value packages usually bid aggressively to win synergistic packages, while avoiding bidding on single items or overlapping packages.

Pikovsky (2008) studied bundle selection strategies using computational experiments for single-item iterative combinatorial auctions. He investigated several bundle selection strategies, including bestResponse, PowerSet, Item bidding[1] and heuristic PowerSet bidding. Item bidders bid only on individual items while bestResponse and PowerSet bidders evaluate all possible bundles. Different types of heuristic PowerSet bidders implement heuristics which might closely resemble real bidders. It was found that bidding on only single items (item bidding) led to loss of efficiency. The PowerSet bidder strategy was found to be best in terms of efficiency outcomes. The auctions with bestResponse bidders achieved significantly lower revenue than other auctions, except for auctions dominated by item biddings. In summary, both real-world observations and previous studies on package selection indicate that alternative bidding strategies need to be considered when evaluating auction performance.

In this study, we consider the following bidder package selection strategies. In each case, the surplus expected for a package is computed by taking the item feedback prices as indicative item prices.

- *bestResponse bidding (BR)*: This strategy is also known as myopic bidding, straightforward bidding or bidding the gradient (Ausubel and Milgrom, 2002). bestResponse bidding has been used by Parkes (2005), Chen and Takeuchi (2005), Parkes and Kalagnanam (2005), Kwon *et al* (2005) and Pikovsky (2008). bestResponse bidders bid 'straightforwardly', offering the package that has the highest expected surplus, which is measured as the gap between the actual cost of the package and its computed value at current (feedback) prices.
- *PowerSet bidding (PS)*: The PowerSet bidder evaluates all possible packages in each round, and submits bids for all packages with positive expected surplus. PowerSet bidding is also known as limited straightforward bidding (Pikovsky, 2008).
- *Constrained PowerSet bidding (CPS)*: In constrained PowerSet bidding, the bidder anchors on the bestResponse package to identify other suitable packages, say, within 50% range of the value of the expected surplus of the anchored package. This strategy will allow bidders to reduce their bidding costs by focusing on a fewer number of packages compared with PowerSet bidders. Similar strategies have been adopted by Ausubel and Milgrom (2002) and Pikovsky (2008).
- *Heuristic PowerSet bidding (HPS)*: HPS bidders randomly select around half of the PowerSet bids. Therefore, while CPS bidders would select a subset of PowerSet bids with the highest expected surplus, HPS bidders might select bids with a low expected surplus. This strategy is appropriate when bidders are either not fully confident about the current market information or are motivated by factors other than profit maximization.
- *Naïve bidding (NV)*: A naive bidder submits bids on all possible combination of items, regardless of expected surplus. Performances of different bundling strategies are often compared against the efficiency achieved by naïve bidders.

[1]In the original paper they refer to this bidding strategy as naïve bidding. But to avoid confusion we refer to it as item bidding since bidders could bid only upon individual items.

When the auction starts, all types of bidders (except naïve bidders) randomly select a subset of packages to start with. After the initial round, bidders rely on the results from the preceding round to revise their choice of packages. For example, provisional winning bidders would resubmit their provisionally winning packages in the following round. Losing bidders would select packages according to their respective package selection strategy.

The bid price revision between rounds relies on the item price feedbacks provided by the auctioneer. The feedback prices are used in an Experience Weighted Attraction (EWA) learning algorithm (Camerer, 2003) to set prices for the next round. The EWA learning allows bidders to compare expected payoff for the full set of pricing strategies before selecting an optimal one. The details of the EWA algorithm are presented in Appendix B.

3.3. Degree of competition

We test the price feedback designs for three different levels of competition. Competition is varied by changing the degree of rationing (DR) which is defined as the percentage of bidders who could supply the target demand in an optimal allocation where contracts are offered to the least cost providers. For example, DR20 means that the target demand has been set in such a way that 20% of the bidders could meet the target. Similarly, DR40 means that 40% of the bidders could satisfy the demand. The higher the percentage of bidders included in the optimal allocation that would satisfy the target demand, the lower the degree of competition. The three competition levels along with the target demand structure are described in Table 1. As a reference, the last column indicates the least cost (socially efficient) way of meeting the target demand. The socially efficient allocation occurs when services are sourced from the most efficient sources and provides a good benchmark for measuring auction allocative efficiency as discussed later.

Table 1 Composition of demand targets used to generate different levels of bidder competition

Degree of rationing (% of bidders in optimal allocation)	*Composition of target demand*			*Optimal cost (million $)*
	Malleefowl	*Phascogale*	*Python*	
DR20 (20)	160	160	16	1.446
DR40 (40)	320	320	32	2.893
DR60 (60)	480	480	48	4.339

In all our computational experiments, we use a population of 20 bidders who are homogeneous in cost structure and bidding strategy. Homogeneity in the bidder population has helped us to concentrate on the performance of the designs under different package selection rules without worrying about the effects of heterogeneity in bidder cost structures. Each auction simulation is run for 200 rounds. This number was chosen because we found that it was long enough for convergence under all schemes. All the simulations have been replicated 200 times to smooth out the effect of randomness in the initial package and price selection strategies. Reported results are based on the averages values from these replications.

3.4. Performance measures

The following three criteria are used to evaluate auction outcomes: allocative efficiency, rent extraction and speed of convergence. Allocative efficiency (AE) refers to the degree to which the cost of procurement is minimized. AE is maximized when the auction selects the least cost sources to meet the target (Pekeč and Rothkopf, 2003). AE is measured as the ratio of the least possible cost to the actual cost of procurement. The degree of rent extraction (RE) estimates the amount of overpayment to the winning bidders. RE measures the proportion in bidder revenue of pure rent (revenue over cost) earned by bidders. Values above zero indicate the presence of rent extraction. The speed of convergence is measured as the number of rounds (Round) required for the RE value to converge on its final level.

4. Results and discussion

We use regression analyses to summarize the implications for the results for the relationship between auction features and performance measures. The analyses relate each of our auction performance indicators (namely AE, RE and Round) to the following three sets of auction features:

(1) auction price feedback designs (represented by dummies for ConsNuc, Nuc, RAD LP, RAD NLP, SmAnch and DEA-based algorithm, with RAD NLP used as the benchmark);
(2) degree of rationing (represented by dummies for DR20, DR40 and DR60, with DR60 used as the benchmark); and
(3) bidder package selection strategy (represented by dummies for BR, CPS, HPS, PS and NV, with BR used as the reference category).

Coefficient estimates for the different outcome regressions are presented in Table 2 and discussed in the following sub-sections starting with the auction efficiency outcomes.

4.1. Auction outcomes: efficiency and rent extraction

In general, efficiency outcomes are considered as the key criteria for measuring performance. The results summarized in the table show that both allocative efficiency (AE) and rent extraction (RE) are significantly affected by price feedback design, bidder package selection strategy and the degree of rationing (or degree of competition). Auctions with lower degrees of rationing (or lower levels of target demand) achieve higher degrees of efficiency outcomes. For example, allocative efficiency is increased by 3.00 percentage points and the rate of rent extraction is reduced by more than 6.00 percentage points when the degree of rationing in an auction is reduced from high (DR60) to low (DR20) (see last two rows of coefficients in Table 2). These indicate that it is more difficult for the algorithms to find an optimal allocation or minimize rents in low competition environments or when the target demand is large.

The coefficient estimates for the price feedback algorithm dummies indicate that auctions based on the SmAnch and ConsNuc algorithms achieve higher allocative efficiency than any other algorithms. This is followed by the performance of DEA-based auction designs. Similarly, the rate of rent extraction was lowest when the auctioneer employed SmAnch- and ConsNuc-based auction designs (Table 2). For example, an auction based on the SmAnch algorithm achieves 0.60 percentage points higher allocative efficiency compared with an auction based on a RAD NLP algorithm. Similarly, the degree of rent extraction will be minimized by 0.60 percentage points if an auctioneer uses a SmAnch algorithm instead of a RAD NLP algorithm. As noted in the appendix (Appendix A), the SmAnch algorithm solves a one-step linear optimization to reduce the total sum of slack variables for the losing bids. Once the total amount of infeasibility is found, it runs a separate optimization to reduce the fluctuations in prices between two consecutive rounds. This reduction in price signal fluctuations might have helped the algorithm to achieve high allocative efficiency since strongly fluctuating item prices can confuse bidders and misguide them in their bid formulation and revision. The ConsNuc algorithm does not have any such ‘price balancing’ optimization. Instead, it allows the slack variables freedom to take either positive or negative signs. Probably, this feature has helped

Table 2 OLS regression results relating auction outcomes to auction features

	AE	*RE*	*Rounds*
(Constant)	0.948*** (0.001)	0.111*** (0.002)	29.403*** (0.864)
Price feedback			
ConsNuc	0.006*** (0.001)	−0.006*** (0.002)	0.016 (0.886)
DEA	0.001 (0.001)	0.003** (0.002)	4.223*** (0.886)
Nuc	−0.002** (0.001)	0.009*** (0.002)	1.634 (0.886)
RADLP	−0.004*** (0.001)	0.012*** (0.002)	1.4 00 (0.886)
SmAnch	0.006*** (0.001)	−0.006*** (0.002)	1.593* (0.886)
Package strategy			
CPS	0.015*** (0.001)	−0.048*** (0.001)	9.987*** (0.619)
HPS	0.012*** (0.001)	−0.044*** (0.001)	31.538*** (0.622)
NV	0.029*** (0.001)	−0.074*** (0.001)	4.331*** (0.616)
PS	0.021*** (0.001)	−0.059*** (0.001)	11.397*** (0.619)
Degree of rationing			
DR20	0.031*** (0.000)	−0.061*** (0.001)	−36.838*** (0.480)
DR40	0.027*** (0.000)	−0.052*** (0.001)	−25.187*** (0.481)
F-statistics	798.772***	1009.702***	840.160***
Adjusted R^2	0.353	0.408	0.364
Number of observations	16 095	16 095	16 095

Note: *** and ** indicate significance at 1 and 5% level of significance, respectively. Numbers in parentheses are standard errors for the respective coefficients.

ConsNuc to explore more pricing options faster and achieve higher allocative efficiency.

The results also show that auction outcomes vary depending on the package selection strategies of the bidder population. The higher the number of packages offered by a bidder type, the better the efficiency outcomes. Allocative efficiency is highest for auctions with naïve bidder (NV) types and this is followed by estimates for auctions with PowerSet bidders (PS), constrained PowerSet (CPS) and Heuristic PowerSet (HPS) bidder types, respectively. Lowest allocative efficiency was achieved in auctions when bidders adopt bestResponse (BR) package selection strategy. For example, an auction with NV bidder type achieves 3.00 percentage points higher allocative efficiency compared with an auction with BR bidder type (Table 2). The results for allocative efficiency mirror those for rent extraction: the degree of rent extraction is lowest for auctions with naïve bidders and this is followed by auctions with PS, CPS and HPS bidder types. Rent payments were highest for auctions where bidders adopted a bestResponse strategy. An auction with NV bidder types minimizes the degree of overpayment by 7.40 percentage points compared with an auction with BR bidder types (Table 2).

Better efficiency outcomes for auctions with naïve bidders suggest that the auctioneer benefits from encouraging bidders to submit bids on all possible combination of bids. However, bidders are less interested in bidding on all possible combinations since profits for winning bidders tend to go down if the bidders adopt this strategy. Moreover, for several reasons, it is not always possible for bidders to evaluate all possible package types. Firstly, the cost of package evaluation and bidding grows exponentially with the increase in the number of packages under consideration. Secondly, it does not make sense to bid on all packages in every round in an iterative auction since bidders know that they will get a chance to select new packages or revise their bids on existing packages in future rounds.

However, power set bidding offers similar benefits without bidders having to work through a large set of bids as in naïve bidding. PowerSet bidders bid only on packages that have a positive expected surplus in any round. Therefore, the strategy leads to a much lower number of packages or offers compared with auctions with naïve bidders. However, since bidders select the packages strategically to extract rent, PowerSet bidding leads to higher rent extraction compared with naïve bidding. Two variants of PowerSet bidding (namely, CPS and HPS) that limit the number of packages or offers perform worse than PowerSet bidding (Figure 2). Therefore, auctioneers might find the promotion of PowerSet bidding interesting.

Bidders might be interested in other approaches. In particular, the rent extraction estimates in Table 2 indicate that winning bidders benefit most by adopting bestResponse (BR) bidding strategy since this strategy offers the best rents. There are several reasons for that. Firstly, in BR bidding, bidders only submit package(s) with the highest expected surplus in order to maximize expected profit. Secondly, in BR bidding only a limited number of

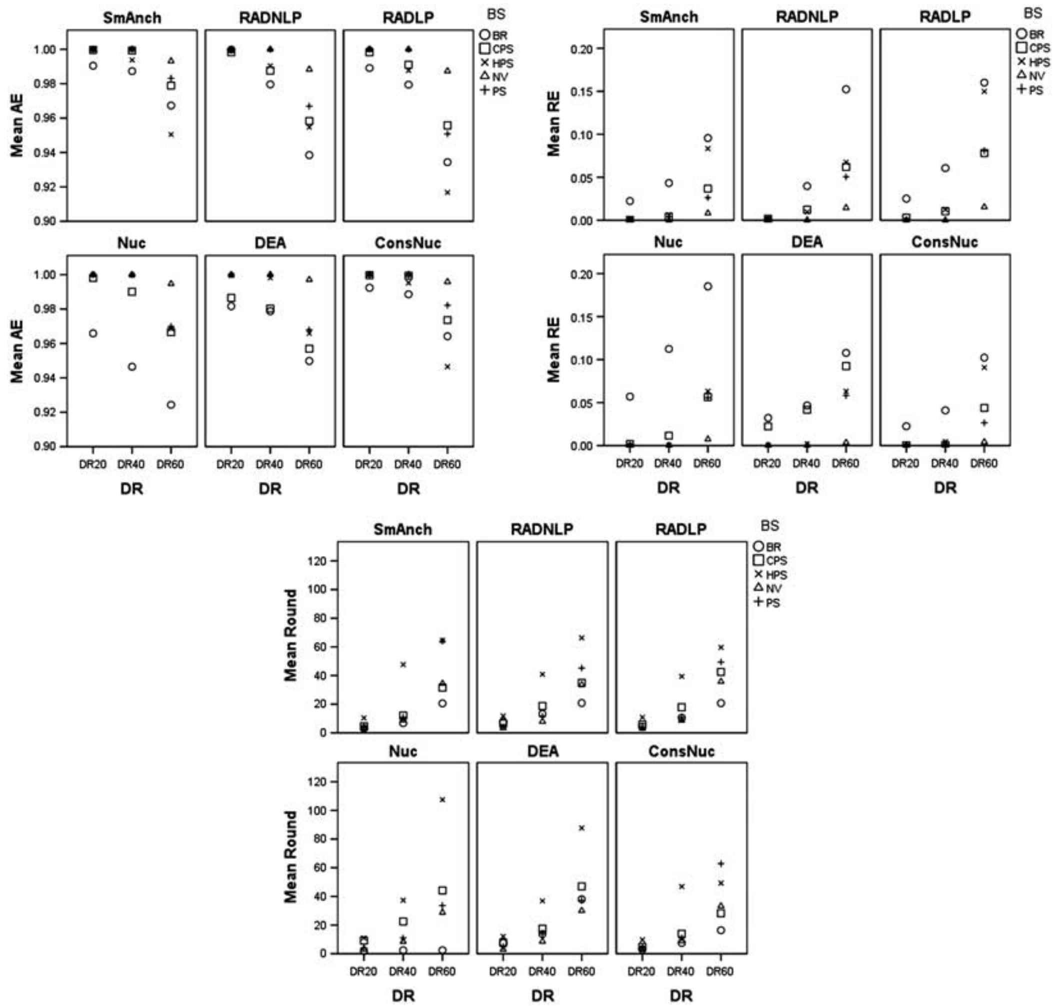

Figure 2 Average auction outcomes (AE, RE and Round) achieved by auction designs for different bidder types (BS) and different degrees of rationing (DR).

bids, in a round, are submitted. For example, compared with PowerSet bidders, bestResponse bidders submit 60% fewer bids, on average. With the submission of only select packages, the auctioneer has a limited set of packages to choose from. Feedback prices are based on these select packages and might not convey information that promotes more competitive bidding in subsequent rounds.

While the general trends discussed above indicate that auction allocative efficiency (rent extraction rates) tend to increase (decrease) with the number of packages offered by a strategy, the outcomes can depend on price feedback algorithms. Efficiency outcomes of some algorithms are more sensitive to particular types of package selection strategy (Figure 2). For example, the negative effect of the use of bestResponse strategy on efficiency outcomes is most acute for auctions based on the Nuc algorithm. In the Nuc algorithm, winning bids are aggregated together to calculate item prices (see Appendix A), whereas computed values for individual winning bids could be below or above their respective bids. With bestResponse bidding, there is a possibility that only a limited number of packages are included in the provisional winning combination. Therefore, there is the possibility that there are large differences between submitted package bid values and package values computed at the price feedback offered by the Nuc algorithm when the BR bidding strategy is employed. These large discrepancies can lead to large swings in the set of packages offered by bidders in the subsequent round. That is, it is more difficult for the Nuc algorithm to guide bidding towards an optimal allocation when bidders use the BR strategy and offer a limited set of packages in any given round.

On the other hand, the DEA-based algorithm has allowed a higher degree of overpayment compared with other algorithms when bidders use the CPS bidding strategy. This is again related to the number of packages submitted under each strategy. In DEA-based algorithms, item prices are calculated in such a way that there is no slack for losing bids and this results in a fewer number of bids with positive surplus in every round compared with other algorithms. Therefore, the CPS bidder submits a fewer number of packages with DEA-based item prices than under other types of item price schemes. This leads to a limited number of packages for the auctioneer to select from. This enables the CPS bidders to extract more rent and the DEA-based auction design to miss the optimal allocation. On the contrary, SmAnch and ConsNuc are less sensitive to changes in package selection strategy compared with other algorithms. Therefore, on average, these algorithms achieve higher efficiency outcomes than other designs.

4.2. Speed of convergence

Simulation results indicate that the speed of convergence for an auction increases with the increase in competition strength (Table 2). For example, an auction with weak competition (ie, DR60) would take 36 rounds more to make a final allocation compared with an auction with strong competition (ie, DR20). The speed of convergence is also affected by bidder package selection strategies. This effect is nonlinear. Convergence is faster if bidders either submit only a few select bids (ie, adopt BR strategy) or submit bids on all possible packages (ie, adopt NV strategy). In other words, the bidding strategy that offers the auctioneer too few choices forces convergence almost as quickly as the strategy which provides the full set of package options in any given auction. In the first case, there is a lock-in effect, with limited choices leading faster to convergence. In the second case, the full set is explored and convergence is as fast as one would expect. On the contrary, auctions with HPS bidder types take longer to reach convergence. HPS bidders not only submit fewer bids than some other types (such as NV and PS) but also rely on a stochastic mechanism to select those packages. Therefore, it takes even longer to reach convergence with this bidder type.

Observing across price feedback algorithms, RAD NLP-based auctions require a lower number of rounds to make final allocation than other algorithms. For example, an auctioneer can save 4.22 rounds making the final allocation using an RAD NLP algorithm instead of a DEA-based algorithm. Most other algorithms take almost the same number of rounds as the RAD NLP to reach convergence (Table 2). However, depending on the competition strength in the auction, some algorithms may take longer to reach convergence. From Figure 2, we observe that in the high and medium competition environments (DR20 and DR40), all algorithms behave similarly in terms of their speed. All algorithms take a higher number of rounds for auctions with HPS bidder type. In a low competition environment (DR60), however, there is a greater variation in speed across the individual algorithms.

5. Conclusion

In iterative multiple unit combinatorial auctions, the selection of packages can be a complex problem for bidders who have to choose from a wide set of packages. Bidders will adopt package selection strategies that are beneficial to them, with implications for auction outcomes. On the other hand, the auctioneer is interested in finding auction designs that perform consistently across different auction environments. Therefore, in this paper we studied the performance of several price feedback computation algorithms for three different levels of competition and for a set of package selection strategies that could be used by bidders. The package selection strategies determine how a bidder chooses which packages to submit in any given auction round and range from a strategy where the bidder focuses on offering a single package in a round (bestResponse) to strategies where all potential packages are offered by the bidder (a naïve strategy).

We find that the performance of an auction design depends on the package selection strategies adopted by bidders. For example, if bidders use a naïve bidding strategy, allocative efficiency is higher than in an auction where bidders are using a bestResponse strategy. Similarly, naïve package selection strategies minimize the degree of rent extraction. Auction convergence, on the other hand, is slowest if bidders adopt a heuristic power set bidding (HPS) strategy which limits offers to a subset of the packages that are likely to generate positive profits for the bidder. The main implication of these findings is that the auctioneer can benefit by promoting bidding strategies that encourage or force bidders to submit more packages for consideration. It might even be cost effective for the auctioneer to provide financial and technical support to bidders so that they are able to prepare bids on multiple projects.

Finally, the auctioneer could readily benefit from selecting a suitable price feedback design. Among the price feedback computation algorithms we studied, smoothed anchoring (SmAnch) and constrained nucleolus (ConsNuc) algorithms performed best in terms of both allocative efficiency and rent extraction rates. Further, the observed efficiency outcomes for these algorithms are relatively robust against changes in the package selection strategies used by bidders. In summary, our results show how an auctioneer could affect auction performance by the use of price feedback designs and by the rules set for bidders with regards to bidding strategies.

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Appendix A

We describe the basic principles of the different price feedback algorithms below. These descriptions are based on Iftekhar *et al* (2011).

A.1. The resource allocation design (RAD)

The RAD of DeMartini *et al* (1999) algorithms first focus on finding a suitable set of slacks, and then on finding an optimal set of prices. Slacks (δ_{ij}) are a measure of the distance between the computed value of a package and its respective bid. (This gap is also called infeasibility gap or duality gap.) There are two variants of the RAD procedure which differ only in whether the slack minimization procedure has a linear or nonlinear objective function.

The linear version of RAD (RAD LP) focuses on the iterative minimization of the maximum of the slack values. Let W and $L = B\backslash W$ be the set of winning and losing bids, respectively. γ^k is the item price for species $k \in K$. In every iteration (t) the maximum of the slack values is minimized so that the computed value for packages in the winning (losing) set of bids are not less than (not greater than) their respective bids. Formally, for the tth iteration of this minimization procedure, we solve the following:

$$\begin{aligned}
&\min \quad z_t \\
&\text{subject to} \\
&\sum_k \gamma_t^k \lambda_{ij}^k \geqslant p_{ij} && \forall j \in W \\
&\sum_k \gamma_t^k \lambda_{ij}^k - \hat{\delta}_{ij} \leqslant p_{ij} && \forall j \in \hat{J}_t = \hat{J}_{t-1} \cup J_{t-1}^*; \\
& && J_{t-1}^* = \{j \in L | z_{t-1}^* = \delta_{ij}^{t-1}\} \qquad \text{(RAD 01)} \\
&\sum_k \gamma_t^k \lambda_{ij}^k - \delta_{ij} \leqslant p_{ij} && \forall j \in L\backslash\hat{J} \\
&0 \leqslant \delta_{ij} \leqslant z_t && \forall j \in L\backslash\hat{J} \\
&\gamma_t^k \geqslant 0
\end{aligned}$$

where γ_t^k is the feedback price for item k; λ_{ij}^k is the number of k items included in the package bid j from bidder i; z_t is the maximum value of a slack (to be minimized); δ_{ij} is the slack for bid j from bidder i; J^* is the set of losing bids whose slacks have been minimized in the previous iteration $(t-1)$; and $\hat{J}$ denotes the subset of L bids whose slacks have been already minimized in all previous iterations $(1, \ldots, t-1)$. After every iteration the set of losing bids with slack values equal to z_t^* are moved into the set J^*. Let $\hat{J} = \hat{J} \cup J^*$ and permanently fix $\hat{\delta}_{ij} = \delta^{*t-1} \forall j \in J^*$. This process continues until one of the following conditions is satisfied: (1) there is no slack in any of the losing bids (ie, $z_t^* = 0$); (2) all the slacks have a value equal to z_t^* (ie, $\delta^* = z^*$); or (3) all losing bids are covered (ie, $\hat{J}_t = L$).

In the nonlinear version of the Resource Allocation Design (hereafter RAD NLP), the objective function to be minimized is simply the sum of the squares of the slack variables (δ_{ij}). Therefore, this version does not involve iterative minimization of slacks.

After finding a suitable set of slacks, both the linear and nonlinear versions run a second linear optimization procedure to iteratively reduce gaps among the item prices. This is done to balance or evenly distribute the prices among the items as far as possible (Goeree and Holt, 2010), which would encourage the participation of smaller bidders or bidders with interests on a subset of items (Kwasnica *et al*, 2005). The price balancing optimization minimizes the maximum item price sequentially while keeping fixed the slack variables obtained from RAD 01. Let $\overline{\delta}_{ij} = \hat{\delta}_{ij}$. Formally,

$$
\begin{aligned}
&\min \quad Y_t \\
&\text{subject to} \\
&\sum_k \gamma_t^k \lambda_{ij}^k \geqslant p_{ij} && \forall j \in W \\
&\sum_k \gamma_t^k \lambda_{ij}^k - \overline{\delta}_{ij} \leqslant p_{ij} && \forall j \in L \\
&\gamma_t^k \leqslant Y_t && \forall k \in K \backslash \hat{K}_t \\
&\gamma_t^k = \hat{\gamma}_t^k && \forall k \in \hat{K}_t = \hat{K}_{t-1} \cup \widetilde{K}_{t-1}; \\
& && \widetilde{K}_{t-1} = \{k \in K \mid Y_{t-1}^* = \gamma_t^{k^*}\}
\end{aligned}
\qquad \text{(RAD 02)}
$$

where K is the full set of item prices; $\hat{K}_t$ is the set of item prices that have already been minimized in all previous iterations $(1, \ldots, t-1)$; $\widetilde{K}_{t-1}$ is the set of item prices minimized in the previous iteration $(t-1)$. After every iteration, the set of minimized prices (ie, $Y^{*t} = \gamma^{k*}$) are moved into a set $\hat{K}_t$. Let $\hat{K}_t = \hat{K}_{t-1} \cup \widetilde{K}_{t-1}$ and kept permanently fixed. The procedure is complete when prices for all items have been sequentially minimized ($\hat{K} = K$). The prices obtained from the final iteration are our desired RAD prices.

A.2. Smoothed anchoring approach (SmAnch)

The smoothed anchoring scheme was first tested by the US Federal Communications Commission (FCC) for spectrum auctions (Kwerel and Rosston, 2000). In the SmAnch algorithm, a two-step procedure is followed. In the first step (SmAnch 01), similar to RAD NLP, a linear optimization is solved to minimize the sum of the slack variables (z^*). Then in the second step a price balancing optimization is solved to reduce fluctuations in prices between rounds (SmAnch 02). Let γ_{r-1}^k be the optimal price obtained in the previous round $(r-1)$ for item k. Formally,

$$
\begin{aligned}
&\min \quad Y \\
&\text{subject to} \\
&\sum_k \gamma_r^k \lambda_{ij}^k \geqslant p_{ij} && \forall j \in W_r \\
&\sum_k \gamma_r^k \lambda_{ij}^k - \delta_{ij} \leqslant p_{ij} && \forall j \in L_r \\
&\sum_i \sum_{\forall j \in L_r} \delta_{ij} = z^* \\
&Y = \sum_k \left(\gamma_r^k - \gamma_{r-1}^k\right)^2 \\
&\delta_{ij} \geqslant 0 \\
&\gamma_r^k, \gamma_{r-1}^k \geqslant 0
\end{aligned}
\qquad \text{(SmAnch 02)}
$$

In summary, this procedure is similar to the RAD NLP procedure other than for the fact that: first, it starts by minimizing the sum rather than the squared sums of slacks; and, second, its second stage item price manipulation involves anchoring prices on previous round item prices while making sure that the item prices rationalize the auction selection results from the previous round as shown in the first two constraints to the above problem.

A.3. Nucleolus-based algorithms

The fourth and fifth algorithms studied for this paper are the nucleolus (Nuc) and constrained nucleolus (ConsNuc) algorithms which were developed by Dunford *et al* (2007) for single unit combinatorial auction. These algorithms do not involve explicit procedures for balancing prices across items as in the RAD or the SmAnch. Instead, these algorithms work simultaneously on an optimal set of slacks and item prices. First, the maximum slack value is reduced sequentially as in RAD. However, unlike in other algorithms, the slack variables are allowed to take any (positive or negative) sign. The absence of sign restriction may generate more flexibility in how slacks and item prices are selected.

The Nuc algorithm has a feature that is a further deviation from the RAD-based designs. In this algorithm, packages in individual winning bids are combined. The computed value of the combined bid is forced to be equal to the minimized cost obtained from the winner determination algorithm (Z), while individual winning bids are free to take computed or implied market values less than or greater than their respective bids. Formally,

$$
\begin{aligned}
&\min \quad z_t \\
&\text{subject to} \\
&\sum_i \sum_j \sum_k \gamma_t^k \lambda_{ij}^k = Z \quad \forall j \in W \\
&\sum_k \gamma_t^k \lambda_{ij}^k - \hat{\delta}_{ij} = p_{ij} \quad \forall j \in \hat{J}_t = \hat{J}_{t-1} \cup J_{t-1}^*; \\
&\qquad J_{t-1}^* = \left\{ j | \sum_k \gamma_{t-1}^k \lambda_{ij}^k - z_{t-1}^* = p_{ij}, \forall j \in L \right\} \quad \text{(Nuc 1)} \\
&\sum_k \gamma_t^k \lambda_{ij}^k - \delta_{ij} \leqslant p_{ij} \quad \forall j \in L \backslash \hat{J}_t \\
&\delta_{ij} \leqslant z_t \\
&\gamma_t^k \geqslant 0
\end{aligned}
$$

After every iteration, we separate the set of bids (J_t^*), for which the computed value for a package minus the optimal slack (z^*) is equal to the respective bid. These bids are then put into a global set $\hat{J}_t; \hat{J}_t = \hat{J}_{t-1} \cup J_{t-1}^*$. In the next iteration, the slack minimization problem is solved for the remaining of the losing bids. The process continues until there is no slack (ie, $z^* = 0$) or all losing bids are covered (ie, $\hat{J}_t = L$).

Similar to RAD LP and Nuc, the ConsNuc algorithm focuses on iterative minimization of maximum slack. However, the ConNuc has the constraint that the computed values of all winning bids are at least as big as their respective submitted bids. In other words, the Nuc is that the first constraint ($\sum_i \sum_j \sum_k \gamma^k \lambda_{ij}^k = Z, \ \forall j \in W$) is replaced with the constraint $\sum_k \gamma^k \lambda_{ij}^k \geqslant p_{ij} \forall j \in W$ in ConsNuc. The rest of the procedure is the same as in the Nuc algorithm.

A.4. Data envelopment analysis-based approach (DEA)

The last algorithm is based on data envelopment analysis (DEA). Aparicio *et al* (2008) have proposed this algorithm for use in multiple unit forward combinatorial auctions. Similar to the Nuc algorithm, the DEA-based procedure aggregates all winning bids but differs in its approach to price computation. Under a DEA scheme, the computed value of the aggregated winning bids (ie, $z = \sum_i \sum_j \sum_k \gamma^k \lambda_{ij}^k \forall j \in W$) is maximized subject to the constraint that, for each losing or winning bid, the package value computed at the item prices being optimized does not exceed the submitted bid value for that package. Therefore, for bids on the efficiency frontier, the computed values are equal to their respective bids, whereas for inefficient bids, the values are below the respective bid. For inefficient bids, which might include winning bids, bidders can use the price information to calculate how much they will have to reduce their bids in order to stay competitive in the following round.

Appendix B

B.1. The EWA learning algorithm

In the EWA learning model, it is assumed that each bidder has a set of 10 pricing strategies. Let s_{ij}^g denote bidder i's gth strategy for pricing a package j. Each strategy defines a multiplicative mark-up factor which is used to determine the value or bid amount associated with that strategy, $v(s_{ij}^g)$. Selection of strategy s_{ij}^1 (ie, a mark up of 1 in our case) in a round means that the bidder will bid her true cost for the package. Thereafter, the value of the mark-up factor is incremented by a factor of 0.1. Therefore, the selection of the last (10th) strategy, s_{ij}^{10}, means that the bidder is bidding 1.9 times of the actual cost for the package. This upper range for mark-up was chosen to cover the full set of plausible strategies (ie, a bidder would not have an optimal strategy that is above 1.9 times of its cost in our case). It has been assumed that in the first round, bidders randomly select any one of the strategies for a package. In subsequent rounds, the bidder uses the learning algorithm as described in Ho *et al* (2008) to determine his/her bidding strategy.

The probability of choosing a pricing or mark-up strategy depends on a numerical attraction $q_{ij}^g(t)$ value assigned to that strategy and the experience weight of the bidder denoted by $N(t)$. All bidders start the auction with a prior experience weight, $N(0)$ and an initial value for the attractions $q_{ij}^g(0)$ measures. The learning model updates these values through the rounds and uses them as the basis for selecting a pricing strategy in a round. Given a set of attractions on the strategy set, the bidder uses a logit rule to generate the probability of choice for the strategies.

It is easier to describe the details of the EWA learning algorithm by reference to the set of rules in it, which are shown in Equation (B.1) from Ho *et al* (2008).

$$
\begin{aligned}
&N(t) = \phi \cdot (1-k) \cdot N(t-1) + 1 \\
&q_{ij}^g(t) = \frac{\phi \cdot N(t-1) \cdot q_{ij}^g(t-1) + \left[\delta + (1-\delta) \cdot I\left(s_{ij}^g, s_{ij}(t-1)\right)\right] \cdot R_{ij}^g}{N(t)} \\
&\text{where} \\
&R_{ij}^g = \begin{cases} v(s_{ij}^g) - v(s_{ij}^1) & \text{if } CV_{ij}(t) \geqslant v(s_i^g) \\ 0 & \text{otherwise} \end{cases} \\
&p_{ij}^g(t+1) = \frac{e^{\lambda \cdot q_{ij}^g(t)}}{\sum_H e^{\lambda \cdot q_{ij}^h(t)}} \qquad \text{(B.1)}
\end{aligned}
$$

In the first equation above, the experience weight is adjusted as a function of a decay parameter, ϕ and a growth controller, k. The parameter ϕ reflects the depreciation of past experience. The higher the value of ϕ, the more the agent 'remembers' past experience. The parameter k controls the rate at which attractions grow. Agents with higher k would lock into a strategy more quickly. The updated experience weight is then used to revise the attractions.

As in reinforcement learning, attractions are updated by adding a decayed lagged attraction to an (expected) payoff for the strategy, experience weights are used to weight lagged attraction and to normalize the new attraction level. In the equation above, the bidder's expected payoff from adopting a strategy s_{ij}^{g} in round t is denoted by R_{ij}^{g}. If the expected payoff relates to a strategy that was actually used (and this is captured by the indicator variable $I(s_{ij}^{g}, s_{ij}(t))$, then the entire payoff is used. However, if it relates to a forgone payoff (and is thus just an indication of a potential payoff), then the value is discounted to reflect this by the parameter δ, which measures the sensitivity of the bidders to foregone payoffs. The bidder will consider the expected payoffs of its strategies, which will allow it to bid below or equal to the market price of the package, $CV_{ij}(t)$.

Finally, as indicated in the last part of the equation, the updated attractions are mapped into a probability function to determine the probability of choosing a strategy, $p_{ij}^{g}(t+1)$, subject to the attraction sensitivity parameter, λ. The higher the sensitivity parameter, the more responsive the bidder would be to the strategy with maximum attraction weight.

Following Iftekhar and Hailu (2012), we have used the following parameter values: 0.50 for ϕ and k and 0.80 for δ and λ. These parameter values are the same for all bidders to ensure that the bidder populations do not vary in terms of their learning in different experiments as this is not the main focus of this paper. It has been observed that selection of these values would enable bidder agents to explore the market before selecting a strategy. All types of bidders keep updating the probabilities of all pricing strategies for all possible packages using the available market information. This has been done to tackle price selection problem in a situation when it becomes profitable to submit a package in an intermediate round which was never submitted before in an auction. As mentioned above the provisional winning bidders are not allowed to change their winning bid prices in the following round, whereas the losing bidders use the EWA learning algorithm to select prices for revised set of packages.

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